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Hypersonic Aerothermoelastic Characteristics of a Finned Missile

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A nonlinear analysis has been performed of the aerothermoelastic characteristics of the fins on a tactical missile operating at small angles of attack at low hypersonic speeds. It is found that the divergence velocity can be overestimated greatly by the use of classical linearized theory. Examples are given of the effects of vehicle geometry and aerodynamic heating on divergence and flutter charactristics. Heating effects, per se, can degrade the aeroelastic characteristics severely. However, the missile nose bluntness, required for thermodynamic survival at hypersonic speed, will produce an increase of divergence and flutter speeds through reduction of the dynamic pressure field at the fins. Viscid-inviscid interaction effects on the fins themselves are small in the operational range investigated, but the viscous interaction between fin and body boundary layer can degrade the aeroelastic stability of the fins appreciably.

Nomenclature

a = speed of sound
c = reference length, airfoil chord
d_N = diameter of axisymmetric nose bluntness
H = altitude
M = Mach number $= U/a$
m_P = pitching moment of an airfoil strip coefficient: $c_m = m_p(\rho_\infty U_\infty{}^2/2)c^2$
n = normal force of an airfoil strip coefficient: $c_n = n/(\rho_\infty U_\infty{}^2/2)c$
q = pitch rate
$\bar{q}$ = dynamic pressure $= \rho U^2/2$
T = temperature
t = time
W_t = lateral load applied to fin tip, lb
x = chordwise distance from leading edge
y/d_N = spanwise fin coordinate (Fig. 3)
α = angle of attack
Δ = increment
θ_w = wedge half-angle
ξ = dimensionless x coordinate $= x/c$

Subscripts

D = divergence
e = boundary-layer edge
F = flutter
i = inviscid flow
v = viscosity effect
w = wall
∞ = freestream

Derivative Symbols

$$\dot{\alpha} = \frac{\partial \alpha}{\partial t} \qquad c_{mq} = \frac{\partial c_m}{\partial (cq/U_\infty)}$$

$$c_{n\alpha} = \frac{\partial c_n}{\partial \alpha} \qquad c_{m\dot{\alpha}} = \frac{\partial c_m}{\partial (c\dot{\alpha}/U_\infty)}$$

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Introduction

WHEN considering the aerothermoelastic characteristics of a finned tactical missile operating in the low hypersonic speed range, which is the subject of the present paper, one encounters several problem areas. The aerodynamic heating causes a degradation of structural properties and, in combination with thermally induced stresses, accentuates nonlinear structural behavior. Aerodynamically, the usual nonlinear airfoil characteristics of the fins are complicated by the nonuniform flowfield generated by the slight nose bluntness necessary for the thermodynamic survival of the missile nose. The objective of the present analysis is to combine these nonlinear features in one computer program so that all of the couplings between aerodynamic, thermal, and structural effects can be accounted for in the prediction of divergence and flutter speeds for the fins on a tactical missile flying at near zero angle of attack at freestream Mach numbers from 3 to 6.

At subsonic and low supersonic speeds, linear theories generally suffice to define both structural and aerodynamic behavior. There are exceptions such as the nonlinear flow behavior at transonic speeds and nonlinearities at all speeds caused by separated flow. Although the aeroelastic stability failure is the result of nonlinear coupling, the equations can be linearized, making it possible to treat the different phases of aeroelastic analyses independently. For this purpose the effects of thermally induced prestress and small geometric imperfections are ignored so that the structural stiffness becomes invariant with time. The removal of time as a variable in the definition of the aerothermoelastic environment results in a greatly simplified analysis.

Once the flight regime is extended to high supersonic speeds, aerodynamic heating must be included and the analysis becomes more complicated. The temperatures vary with time, and since thermal stresses change the structural stiffness, the aeroelastic problem cannot be formulated in terms of the current flight condition even if one assumes linear structural behavior. The classic aeroelastic triangle of interacting inertial, elastic, and aerodynamic forces used by Collar[1] becomes the aeroelastic rectangle shown by Rogers,[2] reproduced here as Fig. 1 with only the strong couplings indicated.

Early aerothermoelastic studies were based on linearized theories with thermal, aerodynamic, or structural effects considered separately.[3-6] Few investigations were devoted to their interactions.[7] With the extension of the flight regime to hypersonic speeds, nonlinear effects become important. The nonlinear aerodynamics are well known.[8] The effects of nonuniform aerodynamic heating on the structural stiffness

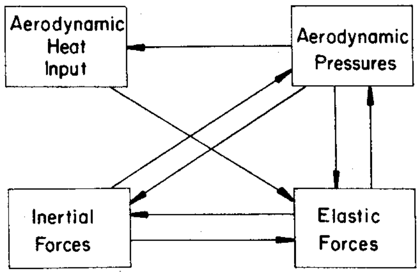


Fig. 1 Aerothermoelastic rectangle (from Ref. 2—arrows denote strong coupling).

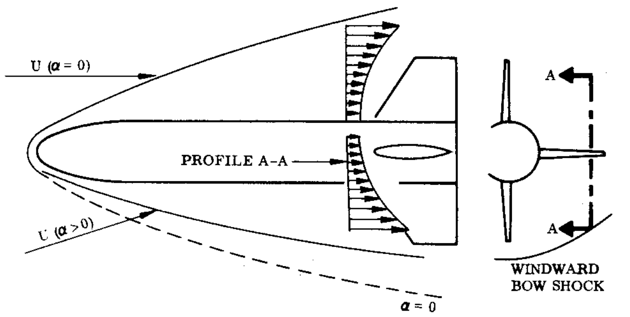


Fig. 2 Hypersonic missile flowfield.

of a missile fin have recently been studied.[9] For thin platelike lifting surfaces, transverse deflections of the same order of magnitude as the plate thickness result in nonlinear force-deflection relationships. It has been possible in the past to evaluate the effects of various complications for greatly simplified mathematical models, but all facets of the aerothermoelastic problem have not been included in one realistic analysis. The various nonlinear interaction can only be accounted for if the aerothermoelastic analysis considers the integrated effect of all the different parts. Such integration is the primary purpose of the present analysis.

Nonlinear Aerodynamics

The flowfield over a typical finned missile is sketched in Fig. 2. A nonuniform inviscid flowfield, the "entropy wake," is generated by the curved bow shock. The dynamic pressure gradients in the "entropy wake" generate highly nonlinear aerodynamic characteristics. For the fins the main effect of the "entropy wake" is to introduce a spanwise dynamic pressure variation. At nonzero angle of attack the effect of the dynamic pressure gradient in the plane normal to the horizontal fins also becomes important (see Fig. 2). Figure 3 shows the dynamic pressure distribution over rectangular fins of unit aspect ratio as determined by embedded Newtonian theory[10-12] in the angle of attack range $0 \leq \alpha \leq 10$ deg. The missile body is a hemisphere-cylinder. The figure illustrates the high degree of nonlinearity in the dynamic pressure profiles. In the present paper only the effect of the spanwise dynamic pressure gradient is considered as the fin is at near zero angle of attack.

It has been documented that viscid-inviscid interaction becomes important at hypersonic speeds, causing large adverse effects on airfoil unsteady aerodynamics.[13] A simple perturbation method was developed to include in the present analysis first-order viscous interaction effects. It has been shown[14] to give predictions that agree well with numerical[13] and experimental results.[15] Based upon this first-order perturbation method, the viscous-induced increase of the normal force derivative for a typical hypersonic fin geometry, a 5-deg half-angle wedge, is as shown in Fig. 4. It can be seen that the viscous-induced effects on the normal force of a fin of 1-ft chord is less than 10% for cold-wall conditions ($T_w/T_e = 1.0$) at 70,000-ft altitude. Even if the maneuvering missile could reach transient soak temperatures as high as those expected on the straight-wing Space Shuttle during reentry,[16] i.e., $T_w/T_e = 2.5$, the viscous-induced increase of the normal force would not exceed 20%. Figure 5 shows the corresponding effect on the static stability. An increase of the normal force derivative by 10 or 20% causes a forward moment of the aerodynamic center of 1.25 or 2.5%, respectively. That is, the viscid-inviscid interaction decreases the static stabililty. The effect on the dynamic stability is shown in Fig. 6. For axis locations§ at which the inviscid aerodynamic effects are statically stabilizing, i.e., for $\xi_{c.g.} < 0.5$, the viscous effect is detrimental also to the dynamic stability. The decrease of dynamic stability is largest for $\xi_{c.g.} = 0.25$, where viscous-induced increases of the normal force derivative of 10 or 20% decrease the dynamic derivative by 10 or 20%, respectively.

The first-order viscous perturbation method[14] was extended to apply also to the aerodynamic effects of elastic deformation of the fin. The results obtained in this manner[14] are in good agreement with numerical results.[17]

The results in Figs. 4-6 indicate that viscid-inviscid interaction of the conventional type will not have an important influence on the aerothermoelastic characteristics of a finned missile operating at low hypersonic speeds and moderate altitudes. However, in addition to the two-dimensional viscid-inviscid interaction, there is a three-dimensional viscid interaction between fin and body boundary layers.[18] Besides causing aggravated heating, this three-dimensional viscous interaction can cause deterioration of the aeroelastic characteristics of a missile fin (Fig. 7). The experimental data[19] show how the "corner flow" between a wedge and the end-plate, used to prevent the tunnel wall boundary layer from interfering, causes dynamic instability if the elastic axis or the rotation center of a rigid fin is located between $\xi_{c.g.} = 0.2$ and $\xi_{c.g.} = 0.4$. Thus, the corner flow effects eliminate practical use of much of the $\xi_{c.g.}$ region corresponding to static stability $\xi_{c.g.} < 0.5$. It is described in Ref. 20 how this "corner flow" problem is aggravated in the case of a tactical missile. Not only is the boundary layer thicker relative to the fin but, more importantly, body crossflow and boundary-layer transition effects increase greatly the impact of the "corner flow" on fin-body aerodynamics. Analytic means have not yet been developed by which these three-dimensional viscous interaction effects can be predicted, and as the two-dimensional effects are completely insignificant in comparison, the inviscid aerodynamic characteristics are used. The airfoil thicknesses and elastic deformations are small, and the present study is restricted to small angles of attack. Consequently, the inviscid unsteady aerodynamics are defined with sufficient accuracy by third-order piston theory.[21]

Aerothermoelastic Characteristics

In the case of a symmetric profile without any geometric imperfections, the divergence velocity at zero angle of attack corresponds to the lowest aerodynamic pressure at which the wing or fin can maintain equilibrium in a slightly twisted configuration. In this case the linear static aeroelastic analysis can be based on an eigenvalue or bifurcation approach. That is, the analysis determines the lowest value of the aerodynamic pressure at which the nonlinear equations of motion possess multiple solutions. A linearized analysis can give the points at which the nonlinear equations have multiple solutions. However, if nonsymmetry, due to nonzero angle of attack, for example, or some finite initial, geometric imperfection is present, there is no bifurcation point. The precritical deformation pattern already contains a component of the displacement mode corresponding to divergence. This

§Either the elastic axis or the rotation center for an all-movable fin.

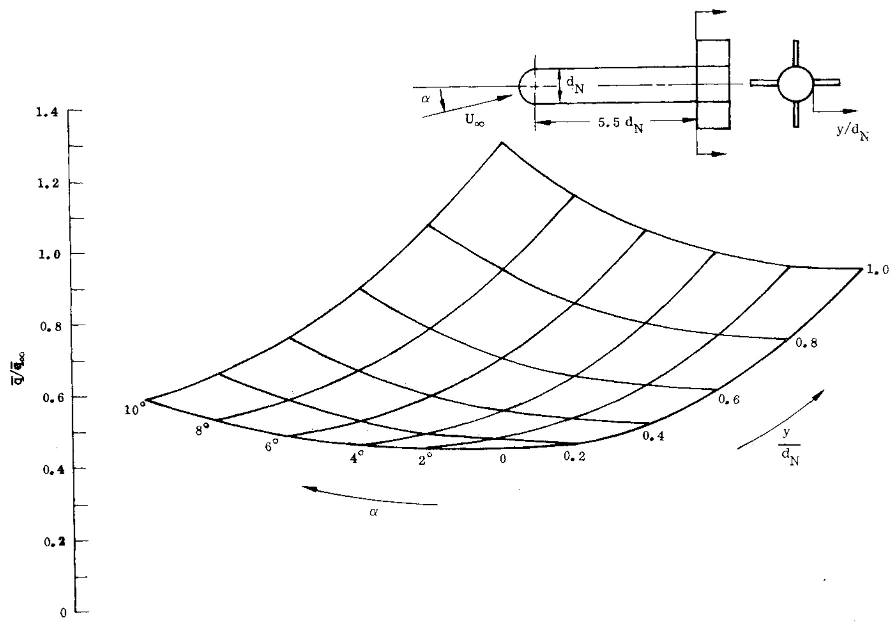


Fig. 3 Dynamic pressure distribution over aft fins on a hemisphere-cylinder.

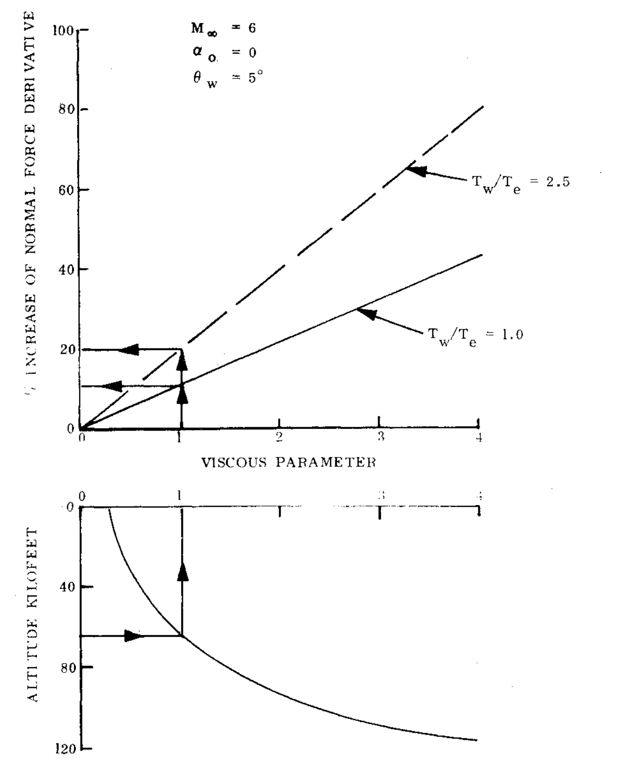


Fig. 4 Viscous effects at $M_\infty = 6$ on the normal force derivative of a 5-deg wedge of 1-ft chord.

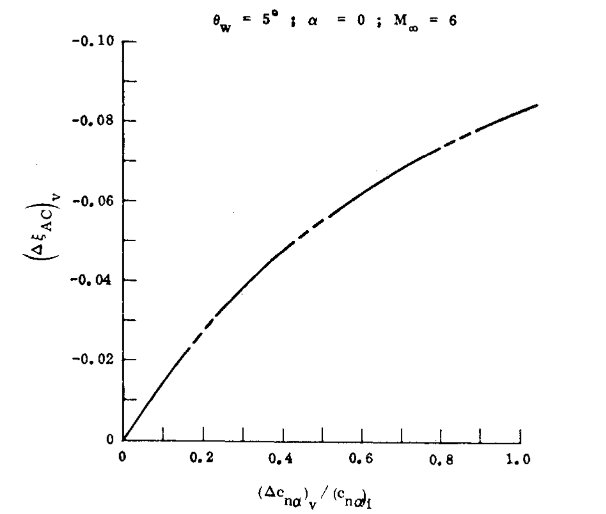


Fig. 5 Viscous effects on the aerodynamic center of a 5-deg wedge at $M_\infty = 6$.

mode will not suddenly appear (at a bifurcation point), but will grow gradually with increasing dynamic pressure. Accurate deformations and stresses in the structure can only be determined from a nonlinear quasistatic solution. Similarly, the linear flutter analysis only establishes boundaries for the velocity range in which self-induced oscillations will occur. In order to determine the rate of growth of the oscillation or the limit cycle amplitude for stationary conditions, it is necessary to integrate the nonlinear differential equations of motion. Consequently, we distinguish in the following between linear and nonlinear static aeroelasticity and between linear and nonlinear elastic vehicle dynamics. The limitations of linearized analyses are evaluated. In addition, the effects of aerodynamic heating, degree of wing clamping, aerodynamic body interference, and wing planform are illustrated.

The aerothermoelastic characteristics of the fins on a missile of the geometry described in Fig. 8 have been determined. The results merely illustrate trends, since the discretization for the structural analysis is somewhat coarse. In Fig. 9, the critical dynamic pressure for divergence of the rectangular wing, as given by classical linear methods, is shown as a function of the angle of attack α deg. As expected, the critical velocity is higher if the effects of the missile body on the flowfield are accounted for. This is due to the nose-bluntness-induced reduction of the ambient dynamic pressure (Fig. 3). The slight decrease of critical velocity (or dynamic pressure) with angle of attack is caused by the nonlinear aerodynamics. In the presence of the missile body this nonlinear trend is amplified by the α-dependence of the nose-bluntness-induced dynamic pressure deficit (Fig. 3). As the aerodynamic force derivatives become insensitive to Mach number at hypersonic speeds,[8] the results in Fig. 9a can be converted directly into a corresponding divergence Mach number M_D, which will vary with altitude. The minimum M_D, obtained at sea level, is shown in Fig. 9b. When the wing is clamped along the entire root chord, instead of being only

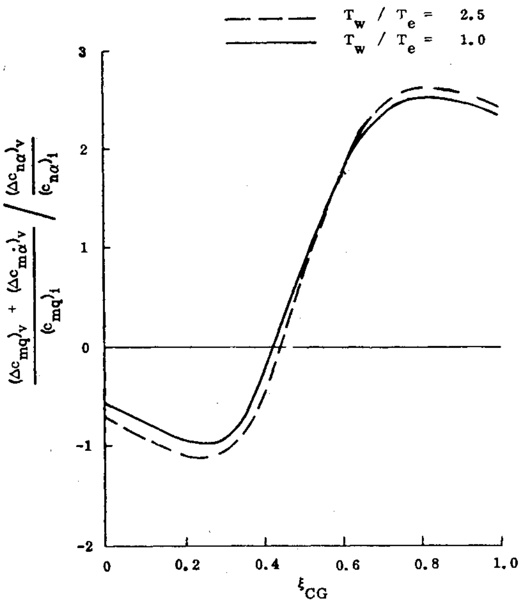


Fig. 6 Viscous effects on the dynamic stability of a 5-deg wedge at $M_\infty = 6$.

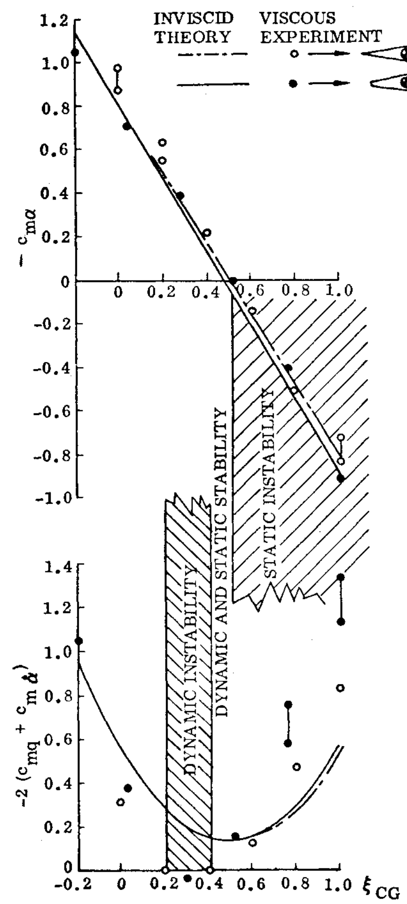


Fig. 7 Stability derivatives of a 9.5-deg wedge at $M_\infty = 9.7$ (Ref. 20).

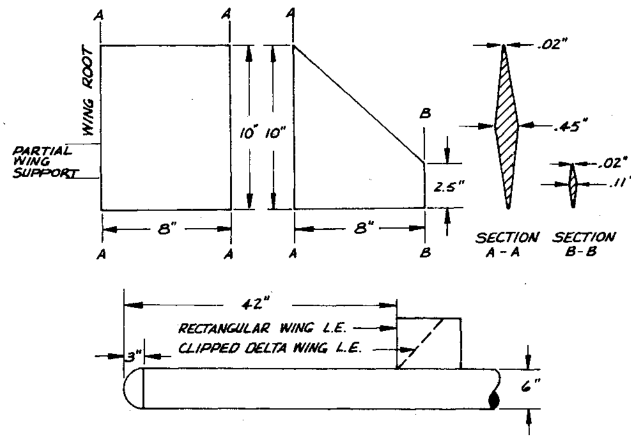
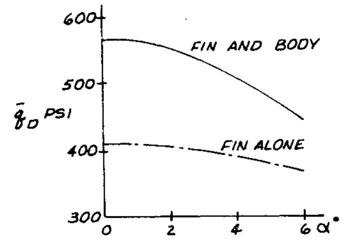
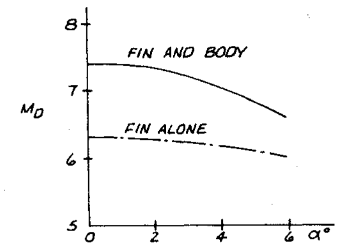


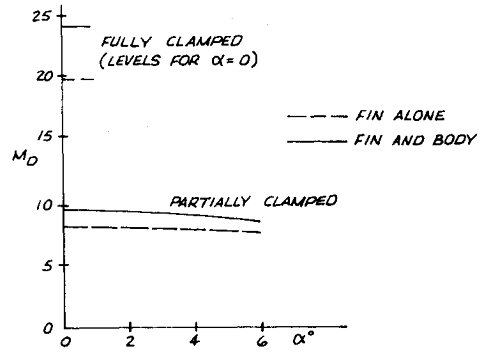
Fig. 8 Missile fin geometry.



a) Divergence dynamic pressure (partially clamped fin)



b) Divergence Mach number ($H = 0$, partially clamped fin)



c) Divergence Mach number ($H = 14{,}000$ ft)

Fig.9 Divergence characteristics of a rectangular fin determined by linear analysis.

partially clamped as indicated in Fig. 8, the divergence speed is increased by a factor of 2.5 (Fig. 9c).

In Fig. 10, the results are shown of a nonlinear divergence analysis for the rectangular wing under isothermal conditions ($\bar{q}_D = 566$ psi in Fig. 9a). A perturbation causing lateral displacements already at low speeds was introduced in the form of a small lateral force at the tip of the leading edge or by specification of a nonzero angle of attack. The stiffness properties of aluminum were used in the analysis. Even for a high-performance alloy the stresses will be well into the inelastic range when the wing tip displacement reaches 0.5 in. Consequently, Fig. 10 shows that for flight at $\alpha = 2$ deg the critical dynamic pressure is no more than one-third of that computed from the classical linear theory, i.e., $M_D = 4.25$ rather than $M_D = 8.0$ for a partially clamped fin (compare Figs. 9c and 10). It should be noted that when the deformation of the wing becomes significant, the total lift will be modified. During flight, the change in total lift due to deformation will be corrected for automatically by a change of the angle of

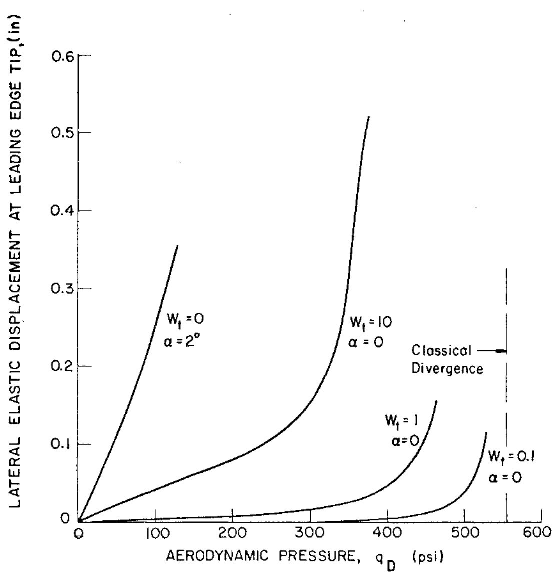
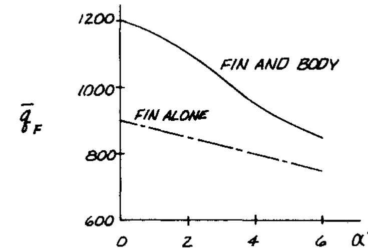
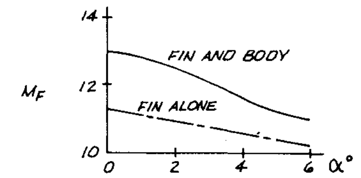


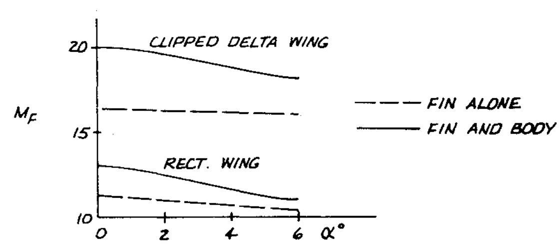
Fig. 10 Divergence characteristics of a rectangular fin at $H = 14{,}000$ ft, determined by nonlinear analysis.



a) Flutter dynamic pressure



b) Flutter Mach number

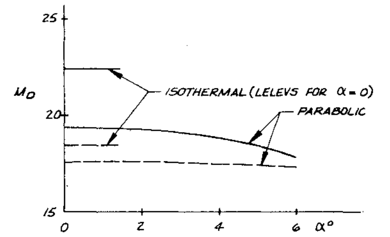


c) Effect of planform

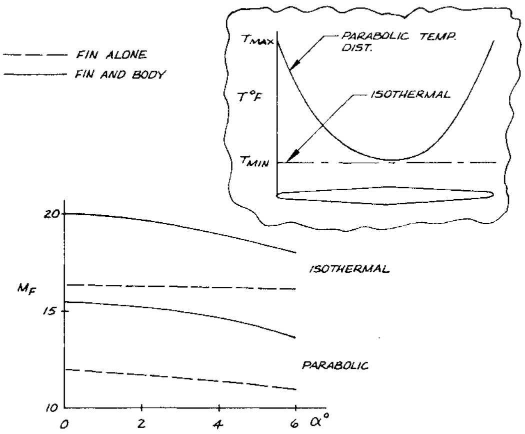
Fig. 11 Flutter characteristics for fully clamped fin at isothermal conditions, $T = 59°F$, at $H = 10{,}000$ ft.

attack of the wing either directly or via a control deflection. No such corrections have been included, and the results shown in Fig. 10 (for fixed angle of attack) are representative of a wind-tunnel test case rather than of controlled flight.

A linear binary flutter analysis of the same rectangular wing gives the critical dynamic pressure shown in Fig. 11a. Again, because the aerodynamic derivatives are insensitive to Mach number for $M_\infty > 5$, the results in Fig. 11a define the flutter Mach number shown in Fig. 11b. The flutter speed decreases with increasing angle of attack, a result that is in agreement with those obtained by other investigators.[22-25] The clipped delta wing planform shown in Fig. 8 was also investigated. Figure 11c shows how the flutter speed increases



a) M_D for $T_{max.} = 450°F$, $T_{min.} = 59°F$



b) M_F for $T_{max.} = 650°F$, $T_{min.} = 59°F$

Fig. 12 Effect of temperature distribution on aerothermoelastic characteristics of fully clamped rectangular fin at $H = 10{,}000$ ft.

when the fin is changed from a rectangular to a clipped delta planform. This increase represents the combined effect of increasing structural stiffness and decreasing aerodynamic loads over the outboard span. The aerodynamic load relief effect is amplified by the presence of the body.

Figure 12 shows the effect of changes in the temperature distribution for the rectangular fin from a constant temperature to a parabolic temperature distribution. It can be seen that both the divergence and flutter speed are decreased (Figs. 12a and 12b, respectively). The decrease of the flutter speed for $T_{max.} = 650$ deg is rather substantial (Fig. 12b), which is in agreement with experimental results.[26] The decrease of the divergence speed is less drastic (Fig. 12a). This is at least in part because the divergence analysis is performed for a lower temperature ($T_{max.} = 450$ deg). It should be noted that in addition to the effect on dynamic pressure shown in Fig. 3 the presence of the body has a similar effect on the ambient temperature field at the fin.[27] While the body-generated spanwise dynamic pressure distribution over the fin is included in the present analysis, the effect of the spanwise temperature distribution is not. Inclusion of this effect, although readily effected, was not deemed to be justified until a more sophisticated thermodynamic analysis becomes available. In the present study a one-dimensional program was used, i.e., heat transfer is considered through-the-thickness only.[28] The neglect of the chordwise heat conduction results in an overestimation of the stresses of the leading and trailing edges. The thermodynamic analysis is the weakest link in the present code for computation of aerothermoelastic characteristics.[12]

The linear flutter analysis indicates only the boundary of the velocity region within which vibration of infinitesimally small amplitudes will grow with time. The complete solution of the nonlinear equations of motion including displacement dependent aerodynamic loads defines at which rate the finite amplitude grows with time. It could show, for example, that a maneuver that includes a short time period at speeds above the flutter speed is safe. Such a nonlinear analysis was performed for the fin alone at $\alpha = 0$ ($M_F = 11.3$ for the rectangular fin in Fig. 11c). The nonlinear transient analysis was started by applying a load of 0.1 lb instantaneously at the tip of the leading edge so that a small-amplitude vibration is initiated.The analysis was then restarted (for the vibrating wing) at flight velocities corresponding to $M_\infty = 0.77\ M_F$, 0.90 M_F, and 1.03 M_F. The results seem to indicate a change to divergent oscillations when the freestream Mach number is increased from $M_\infty = 0.90\ M_F$ to $M_\infty = 1.03\ M_F$, in agreement with the linear flutter results. However, the results are difficult to interpret because the transverse load causes a lack of symmetry in the vibration pattern. In future studies of nonlinear dynamic aeroelasticity, the lateral force should be applied as an impulse rather than as a step function.

Conclusions

A study of the aerothermoelastic characteristics of the fins on a tactical missile operating at near zero angle of attack in the Mach number range $3 \leq M_\infty \leq 6$ has produced an integrated numerical analysis procedure for determination of the static and dynamic linear and nonlinear aerothermoelastic response of sharp-edged, aerodynamically heated, cantilevered lifting surfaces representative of a wide range of tactical missile wings and fins. The program combines a one-dimensional ("through-the-thickness") flat-plate solution for computation of the heat transfer, third-order piston theory aerodynamics modified to account for the nonuniform flow generated by a spherically blunted missile body, and a nonlinear structural analysis program. Important results obtained in the analysis are as follows:

1) The divergence velocity can be overestimated greatly by the use of classical linearized theory.

2) Thermodynamic effects can degrade the aeroelastic characteristics severely, although the effects are less than those indicated by previous analyses for constant thickness plates (see Ref. 9).

3) Missile nose bluntness generates a strong spanwise dynamic pressure gradient over the fin with large impact on the effect of fin planform on aerothermoelastic characteristics. In general, the prevailing effect is the nose-bluntness-induced decrease of the integrated mean dynamic pressure at the fin causing an increase in divergence and flutter speeds.

4) Viscid-inviscid interaction on the fins themselves is not significant for a typical tactical missile operating at moderate altitudes (below 70,000 ft) and hypersonic speeds ($3 \leq M_\infty \leq 6$). However, the three-dimensional flow separation caused by the interaction between the fin and the fuselage boundary layer can cause negative aerodynamic damping of movable fins and degrade greatly the aeroelastic stability of cantilevered fins.

Acknowledgments

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